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B(E2) transition probabilities in the q-rotator model with $SU_q(2)$ symmetry

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Abstract. The $SU_q(2)$ symmetry of the q-rotator model, which for rotational bands predicts squeezing of the energy levels equivalent to the variable moment of inertia (VMI) model, predicts an increase with angular momentum of the B(E2) transition probabilities among these levels, while the rigid rotor model predicts saturation and the interacting boson model (IBM) predicts a decrease. Some evidence supporting the $SU_q(2)$ prediction is presented. The possible usefulness of the quantum algebraic method in extending the VMI concept to B(E2) transition probabilities is pointed out.

Quantum algebras [1–4], which from the mathematical point of view are Hopf algebras as pointed out in [3], are recently attracting much attention in physics, especially after the introduction of the q-deformed harmonic oscillator [5–7]. Initially used for solving the quantum Yang–Baxter equation [8, 9], they have now been used in conformal field theories [10, 11] and in the description of spin chains [12, 13]. The description of squeezed states in terms of q-deformed coherent states [14] has also been attempted [15]. In nuclear physics, the q-rotator model with $SU_q(2)$ symmetry has been successfully used for the description of rotational bands of deformed [16, 17] and superdeformed [18] nuclei, its equivalence to the variable moment of inertia (VMI) model [19] having been demonstrated [17]. The $SU_q(2)$ and $SU_q(1,1)$ symmetries have also been successfully used for the description of rotational [20] and vibrational [21, 22] spectra of diatomic molecules. Much progress has also been made in the q-generalization of the theory of angular momentum [23–30].

The Hamiltonian of the q-rotator model with the symmetry $SU_q(2)$ consists of the second-order Casimir operator of $SU_q(2)$. This Hamiltonian can be rewritten [17] as an expansion in powers of $j(j+1)$. In this form the equivalence of this Hamiltonian to the VMI formula for energy levels becomes apparent. In addition the

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deformation parameter τ (with $q = e^{i\tau}$) acquires a well defined physical meaning, by being connected [17] to the softness parameter of the VMI model.

Although the VMI formalism is very successful in describing energy levels, no extension of it to the description of the B(E2) transition probabilities connecting these levels has been constructed. From the physical point of view, however, if deviations from the rigid-rotator predictions are seen in the energy levels, deviations should also be seen in the B(E2) transition probabilities connecting these levels. The $SU_q(2)$ formalism provides the means for such a connection. The $SU_q(2)$ symmetry predicts a continuous increase of the B(E2) values with angular momentum j, while the rigid rotator predicts saturation at high j and the interacting boson model (IBM) ([31]; for recent overviews see [32, 33]), the successful algebraic model for the description of low-lying collective nuclear spectra in medium- and heavy-mass nuclei, predicts a decrease. It is therefore of interest to check whether this prediction of the $SU_q(2)$ symmetry is supported by the experimental data or not. A first test of this prediction is the subject of this paper.

q-numbers are defined as

$$[x] = \frac{q^x - q^{-x}}{q - q^{-1}} . \tag{1}$$

In the case of $q = e^{\tau}$, where τ is real, they take the form

$$[x] = \frac{\sinh(\tau x)}{\sinh(\tau)} \tag{2}$$

while in the case in which q is a phase ($q = e^{i\tau}$, with τ real) they can be written as

$$[x] = \frac{\sin(\tau x)}{\sin(\tau)} . \tag{3}$$

It is clear that in both cases the q-numbers go to the usual numbers as $q \to 1$ (or $\tau \to 0$).

In the quantum case the generators of $SU_q(2)$ satisfy the commutation relations [5–7]

$$[J_0, J_\pm] = \pm J_\pm \qquad [J_+, J_-] = [2J_0] . \tag{4}$$

The commutation relations take the well known classical form in the limit $q \to 1$.

The irreducible representation (irrep) D^j of highest weight j contains the highest vector $|jj\rangle$ which satisfies the equations

$$J_\pm |jj\rangle = 0 \qquad J_0 |jj\rangle = j|jj\rangle \qquad \langle jj|jj\rangle = 1 . \tag{5}$$

The general basis vector with weight m of this irrep can be constructed through use of the lowering operator J_-

$$|j, m\rangle = \sqrt{\frac{[j+m]!}{[2j]![j-m]!}} (J_-)^{j-m} |j, j\rangle \tag{6}$$

where the q-factorial is defined as

$$[n]! = [n][n-1][n-2]\cdots[2][1]\,. \tag{7}$$

The second-order Casimir operator of $SU_q(2)$ is [5–7]

$$C_2[\mathrm{SU}_q(2)] = J_- J_+ + [J_0][J_0+1] = J_+ J_- + [J_0][J_0-1] \tag{8}$$

for which one has

$$C_2[\mathrm{SU}_q(2)]|j,m\rangle = [j][j+1]|j,m\rangle\,. \tag{9}$$

All of the above equations go to their classical counterparts for $q \to 1$ ($\tau \to 0$).

A q-rotor is a system with Hamiltonian [16–18]

$$H = \frac{1}{2I} C_2[\mathrm{SU}_q(2)] + E_0 \tag{10}$$

where I is the moment of inertia and E_0 is the bandhead energy (for ground-state bands $E_0 = 0$). In the case of q being a phase ($q = e^{i\tau}$) one obtains

$$E_j = \frac{1}{2I}[j][j+1] + E_0 = \frac{1}{2I}\frac{\sin(\tau j)\sin(\tau(j+1))}{\sin^2(\tau)} + E_0. \tag{11}$$

If q is real ($q = e^{\tau}$), the trigonometric functions in (11) should be substituted by hyperbolic ones. It should be noticed that this Hamiltonian, which is a scalar under $SU_q(2)$, is also a scalar under SU(2) [34], and that the quantum number j appearing in (11) is the usual SU(2) angular momentum [34].

By making Taylor expansions of the quantities in the numerator of (11), collecting together the terms containing the same powers of $j(j+1)$ (all other terms cancel out) and finally summing up the coefficients of each power one obtains the following series [17].

$$E_j = E_0 + \frac{1}{2I}\frac{1}{(j_0(\tau))^2}\Big(j_0(\tau)j(j+1) - \tau j_1(\tau)(j(j+1))^2 + \frac{2}{3}\tau^2 j_2(\tau)(j(j+1))^3 - \frac{1}{3}\tau^3 j_3(\tau)(j(j+1))^4 + \frac{2}{15}\tau^4 j_4(\tau)(j(j+1))^5 - \cdots\Big) \tag{12}$$

where $j_n(\tau)$ are the spherical Bessel functions of the first kind [35]. For small values of τ one can further expand the spherical Bessel functions appearing in (12). Keeping only the lowest-order term in each expansion one obtains

$$E_j = E_0 + \frac{1}{2I}\Big((j(j+1)) - \frac{\tau^2}{3}(j(j+1))^2 + \frac{2\tau^4}{45}(j(j+1))^3 - \frac{\tau^6}{315}(j(j+1))^4 + \frac{2\tau^8}{14175}(j(j+1))^5 - \cdots\Big)\,. \tag{13}$$

This result is of the form

$$E_j = E_0 + Aj(j+1) + B(j(j+1))^2 + C(j(j+1))^3 + D(j(j+1))^4 + \cdots \tag{14}$$

which is the expansion in terms of powers of $j(j+1)$ used for fitting experimental rotational spectra [36]. Empirically it is known that the coefficients A, B, C, D, ... have alternating signs, starting with A positive. In addition, each coefficient is smaller than the preceding one by about three orders of magnitude.

Equation (11) has been found suitable for the description of rotational spectra [16, 17] with τ around 0.03. The pattern of alternating signs is already present in (13). For τ around 0.03 the order of magnitude of the coefficients is also the correct one, since each term contains a factor τ^2 more than the previous one. For τ in the region of 0.03, τ^2 is of the order of 10^{-3}, as it should be. It is clear therefore that (13) is suitable for fitting rotational spectra, since its coefficients have the same characteristics as the empirical coefficients of (14). Examples of fits and parameter values are given in [16, 17]. In all cases the fits are of very good accuracy.

The same kind of expansion can be obtained in the framework of the VMI [19] model. In this model the levels of the ground-state band are given by

$$E_j = \frac{j(j+1)}{2\Theta(j)} + \frac{1}{2}C(\Theta(j) - \Theta_0)^2 \tag{15}$$

where C and Θ_0 are the two free parameters of the model, the latter being the ground-state moment of inertia. The moment of inertia for each j is determined from the variational condition

$$\left.\frac{\partial E_j}{\partial \Theta(j)}\right|_j = 0\,. \tag{16}$$

One can obtain [17] the following expansion for the energy:

$$E_j = \frac{1}{2\Theta_0}\left(j(j+1) - \frac{1}{2}\frac{(j(j+1))^2}{2C\Theta_0^3} + \frac{(j(j+1))^3}{(2C\Theta_0^3)^2} - 3\frac{(j(j+1))^4}{(2C\Theta_0^3)^3} + \cdots\right). \tag{17}$$

It is known [19] that C and Θ_0 take on positive values, while

$$\sigma = \frac{1}{2C\Theta_0^3} \tag{18}$$

is the softness parameter, which for rotational nuclei is of the order of 10^{-3} [19]. Thus the coefficients of the expansion of (17) have the proper signs and orders of magnitude.

Comparing (13) and (17) we see that both expansions have the same form. The moment of inertia parameter I of (13) corresponds to the ground-state moment of inertia Θ_0 of (17). The small parameter of the expansion is τ^2 in the first case, while it is the softness parameter $1/(2C\Theta_0^3)$ in the second. When these formulae are used for fitting experimental data, the agreement between $1/(2I)$ and $1/(2\Theta_0)$ is very good, as is the agreement between τ^2 and σ. Therefore the extra parameter of the $SU_q(2)$ model (the deformation parameter τ) turns out to have a well defined physical meaning, by being related to the softness parameter of the VMI model.

It should be noted that for real q the coefficients of the expansion similar to (13) are all positive, so that the corresponding spectrum increases more rapidly than the $j(j+1)$ rule, thus being unable to fit the experimental data, in which an increase slower than the $j(j+1)$ rule is observed.

The stretching effect present in rotational energy levels, which can equally well be described in terms of the VMI model and the $SU_q(2)$ symmetry, should also manifest itself in the B(E2) transition probabilities among these levels. If deviations from the SU(2) symmetry are observed in the energy levels of a band, relevant deviations should also appear in the B(E2) transitions connecting them. In the case of the VMI model no way has been found for making predictions for the B(E2) transition probabilities connecting the levels of a collective band. The $SU_q(2)$ symmetry naturally provides such a link. In rotational bands one has [37]

$$B(\mathrm{E2}: j+2 \to j) = \frac{5}{16\pi} Q_0^2 |C_{k,0,-k}^{j+2,2,j}|^2 \qquad (19)$$

i.e. the B(E2) transition probability depends on the Clebsch–Gordan coefficient of SU(2), while Q_0^2 is the intrinsic electric quadrupole moment and k is the projection of the angular momentum j on the symmetry axis of the nucleus in the body-fixed frame. For $k = 0$ bands one then has [37]

$$B(\mathrm{E2}: j+2 \to j) = \frac{5}{16\pi} Q_0^2 \frac{3}{2} \frac{(j+1)(j+2)}{(2j+3)(2j+5)} \qquad (20)$$

which gives the well known saturation of the B(E2) values with increasing j.

In the case of the SU(3) limit of the IBM, which is the limit applicable to deformed nuclei, the corresponding expression is [31]

$$B(\mathrm{E2}: j+2 \to j) = \frac{5}{16\pi} Q_0^2 \frac{3}{2} \frac{(j+1)(j+2)}{(2j+3)(2j+5)} \frac{(2N-j)(2N+j+3)}{(2N+3/2)^2} \qquad (21)$$

where N is the total number of bosons. Instead of saturation one then gets a decrease of the B(E2) values at high j, which finally reach zero at $j = 2N$. This is a well known disadvantage of the simplest version of the model (IBM-1) due to the small number of collective bosons—s ($j = 0$) and d ($j = 2$)—taken into account. It can be corrected by the inclusion of higher bosons (g ($j = 4$), i ($j = 6$), etc), which approximately restore saturation (see [32, 33] for a full list of references).

Another way to avoid the problem of decreasing B(E2)s in the SU(3) limit of the IBM at high j is the recently proposed [38] transition from the compact $SU(3)$ algebra to the non-compact SL(3,R) algebra. The angular momentum at which this transition takes place is fitted to experiment. In this way an increase of the B(E2) values at high j is predicted, which agrees well [38] with the experimental data for ^{236}U.

In order to derive a formula similar to (20) in the $SU_q(2)$ case, one needs to develop an $SU_q(2)$ angular momentum theory. Irreducible tensor operators for the $SU_q(2)$ algebra are already defined [23–28], and the q-deformed version of the Wigner–Eckart theorem, needed in the derivation of the q-generalization of (19), is also known [29, 30]. In addition, the q-deformed versions of Clebsch–Gordan coefficients, 3-j symbols, 6-j symbols and their inter-relations are known [23–30]. It turns out that an equation similar to (19) holds in the q-deformed case, the only difference being that the Clebsch-Gordan coefficient of the $SU_q(2)$ algebra must be used instead. These coefficients have the form [24, 28]

$${}_qC_{k,0,-k}^{j+2,2,j} = q^{2k} \sqrt{\frac{[3][4][j-k+2][j-k+1][j+k+1][j+k+2]}{[2][2j+2][2j+3][2j+4][2j+5]}} \qquad (22)$$

where q-numbers are defined as before. For $k = 0$ bands one then has

$$B_q(\mathrm{E2}: j+2 \to j) = \frac{5}{16\pi} Q_0^2 \frac{[3][4][j+1]^2[j+2]^2}{[2][2j+2][2j+3][2j+4][2j+5]} . \tag{23}$$

For $q = \mathrm{e}^{\mathrm{i}\tau}$ this equation takes the form

$$\begin{aligned} B_q(\mathrm{E2}: j+2 \to j) &= \frac{5}{16\pi} Q_0^2 (\sin(3\tau)\sin(4\tau)(\sin(\tau(j+1)))^2(\sin(\tau(j+2)))^2) \\ &\quad \times (\sin(2\tau)\sin(\tau)\sin(\tau(2j+2))\sin(\tau(2j+3)) \\ &\quad \times \sin(\tau(2j+4))\sin(\tau(2j+5)))^{-1} . \end{aligned} \tag{24}$$

Before attempting any comparison to experimental data, it is useful to get an idea of the behaviour of this expression as a function of j, especially for the small values of τ found appropriate for the description of ground-state spectra. Expanding all functions and keeping corrections of the leading order in τ only, one has

$$B_q(\mathrm{E2}: j+2 \to j) = \frac{5}{16\pi} Q_0^2 \frac{3}{2} \frac{(j+1)(j+2)}{(2j+3)(2j+5)} \left(1 + \frac{\tau^2}{3}(6j^2 + 22j + 12)\right) \tag{25}$$

We see that the extra factor, which depends on τ^2, contributes an extra increase with j, while the usual SU(2) expression reaches saturation at high j and the IBM even predicts a decrease.

Is there any experimental evidence for such an increase? In order to answer this question one should discover cases in which the data will be consistent with the $\mathrm{SU}_q(2)$ expression but inconsistent with the classical SU(2) expression. (The opposite cannot happen, since the classical expression is obtained from the quantum expression for the special parameter value $\tau = 0$.) Since error bars of B(E2) values are usually large, in most cases both symmetries are consistent with the data. One should expect the differences to show up more clearly in two cases:

(i) In rare-earth nuclei not very much deformed (i.e. with an $R_4 = E_4/E_2$ ratio around 3.0). These should be deformed enough so that the $\mathrm{SU}_q(2)$ symmetry will be able to describe them having, however, at the same time values of τ not very small. Since in several of these nuclei backbending (or upbending) occurs at $j = 14$ or 16, one can expect only 5 or 6 experimental points with which to compare the theoretical predictions.

(ii) In the actinide region no backbending occurs up to around $j = 30$, so that this is a better test ground for the two symmetries. However, most nuclei in this region are well deformed, so that small values of τ should be expected, making the distinction between the two theoretical predictions difficult.

A few characteristic examples (four rare earths and an actinide) are given in tables 1–3. In all cases a least-square fit was obtained, the quality of which is measured by

$$\sigma = \sqrt{\frac{1}{n} \sum_{j_{\min}}^{j_{\max}} (B(\mathrm{E2}: j+2 \to j)_{\exp} - B(\mathrm{E2}: j+2 \to j)_{\mathrm{th}})^2} \tag{26}$$

Table 1. $B(E2: j \to j-2)$ transition probabilities (in Weisskopf units) for ^{152}Sm and ^{154}Gd. Data were taken from [39] and [40] respectively. The quantities in parentheses indicate uncertainties. The $SU_q(2)$ fit was obtained using (24). σ is given by (26).

j	^{152}Sm exp	^{152}Sm $SU_q(2)$	^{152}Sm SU(2)	^{154}Gd exp	^{154}Gd $SU_q(2)$	^{154}Gd SU(2)
2	143 (3)	142	166	157 (2)	163	187
4	209 (3)	210	237	245 (8)	240	267
6	245 (5)	246	261	284 (14)	279	295
8	285 (14)	280	273	311 (17)	315	308
10	320 (30)	322	280	361 (33)	358	317
A		707	829		810	936
τ		0.039	0.0		0.037	0.0
σ		2.42	25.50		4.73	26.44
R_4	3.009			3.015		

Table 2. Same as table 1, but for ^{184}W and ^{188}Os. Data were taken from [41] and [42].

j	^{184}W exp	^{184}W $SU_q(2)$	^{184}W SU(2)	^{188}Os exp	^{188}Os $SU_q(2)$	^{188}Os SU(2)
2	119 (3)	106	137	76 (3)	75	92
4	160 (14)	160	196	121 (6)	113	131
6	183 (10)	195	216	131 (9)	134	144
8	238 (21)	235	226	146 (24)	156	151
10	292 (60)	292	232	190 (50)	185	155
A		524	686		374	458
τ		0.050	0.0		0.044	0.0
σ		7.97	35.89		6.40	18.78
R_4	3.274			3.083		

where n is the number of points used in the fit. The overall scale $A = 5Q_0^2/(16\pi)$ has been treated as a free parameter in both $SU_q(2)$ and SU(2), while in $SU_q(2)$ the deformation parameter τ was also free. In all cases it is clear that the $SU_q(2)$ curve follows the experimental points, while the SU(2) curve has a different shape which cannot be forced to go through all the error bars. Two examples of nuclei with $B(E2)$ values consistent with both symmetries (although $SU_q(2)$ gives better fits than SU(2)) are given in table 4. In table 5 we also give the parameter values obtained from fitting the energy levels of the ground state bands of the nuclei appearing in tables 1–4 using the $SU_q(2)$ formula (11). Several comments are now relevant:

(i) For a given nucleus the value of the parameter τ obtained from fitting the $B(E2)$ values among the levels of the ground-state band should be equal to the value obtained from fitting the energy levels of the ground-state band. In table 5 it is clear that the two values are similar, although in most cases the value obtained from the $B(E2)$s is smaller than that obtained from the spectra. It should be taken into account, however, that in most cases the number n' of levels fitted is different to (larger than) the number n of the $B(E2)$ values fitted. In the single case (^{184}W) in which $n = n'$, the two τ values are almost identical, as they should be.

(ii) One can certainly try different fitting procedures. Using the value of τ

Table 3. Same as table 1, but for ^{236}U. Data from [43]. The $j = 28$ point was not taken into account in the fits, because of its large uncertainty. The $j = 10$ and $j = 16$ points were not taken into account in the determination of the parameters in the SU$_q$(2) case, but were included in the calculation of σ.

j	^{236}U exp	^{236}U SU$_q$(2)	^{236}U SU(2)	j	^{236}U exp	^{236}U SU$_q$(2)	^{236}U SU(2)
2	246 (10)	224	272	16	380 (40)	473	479
4	348 (22)	323	389	18	490 (50)	501	482
6	380 (21)	361	428	20	510 (80)	533	485
8	390 (40)	385	448	22	520 (120)	571	487
10	360 (40)	406	460	24	660 (130)	615	489
12	410 (70)	426	469	26	670 (190)	666	491
14	450 (50)	448	475	28	1100 (500)	728	492
A		1119	1361			1119	1361
τ		0.019	0.0			0.019	0.0
σ		36.97	85.46			36.97	85.46
R_4	3.304				3.304		

Table 4. Same as table 1, but for ^{166}Hf and ^{168}Hf. Data were taken from [44] and [45].

j	^{166}Hf exp	^{166}Hf SU$_q$(2)	^{166}Hf SU(2)	j	^{168}Hf exp	^{168}Hf SU$_q$(2)	^{168}Hf SU(2)
2	128 (8)	131	142	2	154 (8)	141	158
4	197 (15)	191	203	4	208 (23)	205	226
6	205 (30)	217	223	6	237 (24)	234	249
8	250 (120)	237	234	8	250 (25)	256	261
10	254 (180)	258	240	10	260 (40)	280	268
12				12	320 (111)	307	273
A		655	709			703	792
τ		0.028	0.0			0.029	0.0
σ		8.58	14.28			11.44	22.06
R_4	2.965				3.110		

obtained from the B(E2) values for fitting the spectrum one gets a reasonably good description of it, although the squeezing of the spectrum is not as much as it should have been (with the exception of ^{184}W). Using the value of τ obtained from the spectrum for fitting the B(E2) values one obtains an increase more rapid than the one shown by the data (again with the exception of ^{184}W). One can also try to make an overall fit of spectra and B(E2)s using a common value of τ. Then both the squeezing of the spectrum and the rise of the B(E2)s can be accounted for reasonably well although not exactly. One should notice, however, that the experimental uncertainties of the B(E2)s are much higher than the uncertainties of the energy levels.

(iii) Concerning energy levels, the rigid-rotor model and the SU(3) limit of the IBM predict a $j(j+1)$ increase, while the SU$_q$(2) model and the VMI model predict squeezing, which is seen experimentally.

(iv) Concerning the B(E2) values, the VMI model makes no prediction, the rigid rotor predicts saturation at high j, the SU(3) limit of the IBM predicts a decrease, while the SU$_q$(2) model predicts an increase. The evidence presented in this work

Table 5. Parameters of least-square fits of levels of the ground-state bands of the nuclei appearing in tables 1–4 using the q-rotator with symmetry $SU_q(2)$. Data are taken from the same references as in tables 1–4 [39–45]. Equation (11) has been used, with $E_0 = 0$. n' is the number of levels fitted ($n' = 8$ means that the levels with $j = 2, 4, \ldots, 16$ were included in the fit). I (in keV) is the moment of inertia parameter, while τ' is the value of the deformation parameter obtained from fitting the energy levels. σ (in keV) for these fits is calculated by a formula similar to (26). To facilitate comparison, the number n of B(E2) values fitted for each nucleus and the corresponding deformation parameter τ obtained from fitting the B(E2) values are also included.

Nucleus	n'	$1/(2I)$	τ'	σ	n	τ
^{152}Sm	8	16.6	0.057	29.56	5	0.039
^{154}Gd	8	17.0	0.058	28.70	5	0.037
^{184}W	5	18.4	0.048	1.37	5	0.050
^{188}Os	6	23.9	0.071	10.22	5	0.044
^{236}U	15	7.1	0.029	21.74	13	0.019
^{166}Hf	9	20.5	0.062	59.13	5	0.028
^{168}Hf	8	17.7	0.056	31.52	6	0.029

supports the $SU_q(2)$ prediction, but clearly much more work, both experimental and analytical, is needed before final conclusions can be drawn. The modified SU(3) limit of IBM described in [38] also supports the increase of the B(E2) values at high j.

(v) Since E_j has to be an increasing function of j, it is clear from (11) that the condition

$$\tau(j+1) \leqslant \pi/2 \tag{27}$$

must be fulfilled. This is in fact the case in both the rare-earth and the actinide regions. As seen in table 5, in the case of ^{152}Sm one has $\tau' = 0.057$, which implies $j \leqslant 26$, this limiting value being higher than the highest observed j in ground-state bands in the rare-earth region. Similarly, for ^{236}U one has $\tau' = 0.029$, which requires $j \leqslant 52$, this limiting value again being higher than the highest observed j in ground-state bands in the actinide region.

In summary, the $SU_q(2)$ symmetry, which is equivalent to the VMI model as far as the description of rotational spectra is concerned, gives definite predictions for the B(E2) transition probabilities among levels of rotational bands. While the rigid rotor predicts saturation of the B(E2) values at high j and the IBM predicts a decrease, the $SU_q(2)$ symmetry predicts an increase, similar to the one predicted by a recently proposed [38] modification of the SU(3) limit of the IBM. A few examples supporting the $SU_q(2)$ prediction have been presented, although much more experimental information and analytical work is needed before final conclusions can be reached. The main gain from the use of the $SU_q(2)$ symmetry is that it provides a mathematical way for extending the VMI idea to B(E2) transition probabilities. It should be remembered, however, that the numerical coefficients in the $SU_q(2)$ series describing energy levels (13) are not exactly equal to their counterparts in the corresponding VMI series (17), although the powers of $j(j+1)$ and the powers of the small parameter in corresponding terms are the same. This might indicate that the present $SU_q(2)$ symmetry is not necessarily the optimal symmetry for describing the VMI effect and extending it to the description of B(E2) values. It would have been interesting to construct a symmetry giving exactly the same expansion for the energy as the VMI model. The recently introduced method of generalized deformed oscillators [46] might be of interest in this respect. Work in this direction is in progress.

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